MENIIT
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JEE MAIN-2022

COMPUTER BASED TEST (CBT)

DATE : 26-06-2022 (EVENING SHIFT) | TIME : (3.00 PM to 6.00 PM)

Duration 3 Hours | Max. Marks : 300

QUESTIONS & SOLUTIONS

PART A : PHYSICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **Only One** is correct.

1. The dimension of mutual inductance is

(A) $[ML^2 T^{-2} A^{-1}]$ (B) $[ML^{-3} T^{-3} A^{-1}]$ (C) $[ML^2 T^{-2} A^{-2}]$ (D) $[ML^2 T^{-3} A^{-2}]$

Ans. **C**

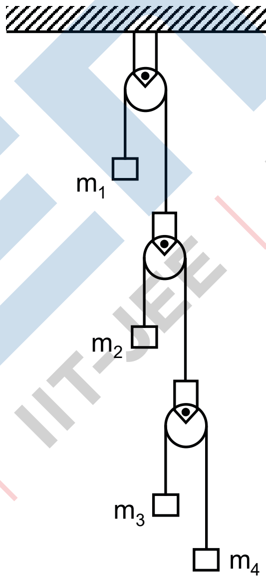
Sol. M = ?

$\phi = Mi = BA$ $F = q v B$

$$M = \frac{A \times F}{i \times qV}$$

$$= \frac{L^2 \times MLT^{-2}}{A \times AT \times L^{-1}} = ML^2T^{-2}A^{-2}.$$

2. In the arrangement shown in figure a_1, a_2, a_3 and a_4 are the acceleration of masses m_1, m_2, m_3 and m_4. Respectively. Which of the following relation is true for this arrangement ?

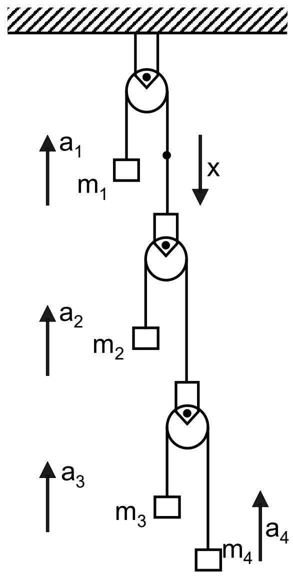


(A) $4a_1 + 2a_2 + a_3 + a_4 = 0$ (B) $a_1 + 4a_2 + 3a_3 + a_4 = 0$

(C) $a_1 + 4a_2 + 3a_3 + 2a_4 = 0$ (D) $2a_1 + 2a_2 + 3a_3 + a_4 = 0$

Ans. **A**

Sol. $a_1 = \dfrac{x - a_2}{z}$



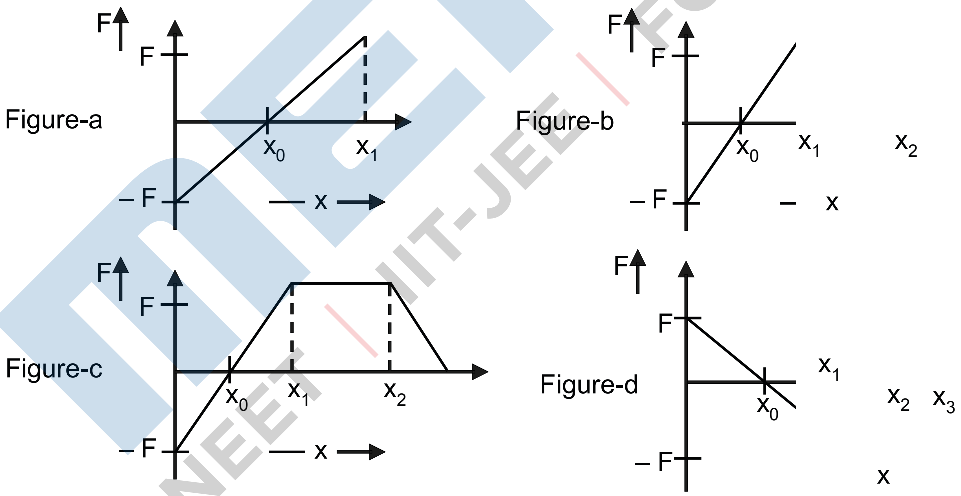
$x = xa_1 + a_2$

$$x = \frac{-a_3 - a_4}{z} = za_1 + a_2$$

$-a_3 - a_4 = 4a_1 + 2a_2$

$4a_1 + 2a_2 + a_3 + a_4$.

3. Arrange the four graphs in descending order of total work done; where W_1, W_2, W_3 and W_4 are the work done corresponding to figure a, b, c and d respectively :



(A) $W_3 > W_2 > W_1 > W_4$ (B) $W_3 > W_2 > W_4 > W_1$

(C) $W_2 > W_3 > W_4 > W_1$ (D) $W_2 > W_3 > W_1 > W_4$

Ans. A

Sol. Positive work greater than negative work

$W_3 > W_2 > W_1 > W_4$.

4. A solid spherical ball is rolling on a frictionless plane surface about its axis of symmetry. The ratio of rotational kinetic energy of the ball to its total kinetic energy is -

(A) $\frac{2}{5}$ (B) $\frac{2}{7}$ (C) $\frac{1}{5}$ (D) $\frac{7}{10}$

Ans. B

Sol. $\frac{KE_R}{KE} = \frac{\frac{1}{2}mV^2 \times \frac{K^2}{R^2}}{\frac{1}{2}mV^2\left(1+\frac{K^2}{K^2}\right)} = \frac{\frac{2}{5}}{7/5} = \frac{2}{5}$

For solid ball $\frac{K^2}{R^2} = \frac{2}{5}$.

5. Given below are two statement : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : if we move from poles to equator, the direction of acceleration due to gravity of earth always points towards the center of earth without any variation in its magnitude.

Reason R : At equator, the direction of acceleration due to the gravity to towards the center of earth.

In the light of above statements, choose the correct answer from the option given below

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true

Ans. D

Sol. Acceleration always directed toward centre

$g_{pole} > g_{equator}$.

6. If p is the density and η is coefficient of viscosity of fluid which flows with a speed v in the pipe of diameter d, the correct formula for Reynolds number R_e is :

(A) $R_e = \frac{\eta d}{\rho v}$ (B) $R_e = \frac{\rho v}{\eta d}$ (C) $R_e = \frac{\rho v d}{\eta}$ (D) $R_e = \frac{\eta}{\rho v d}$

Ans. C

Sol. $R = \frac{\rho v d}{\eta}$

$T = mL^{-3}$

$V = LT^{-1}$

$d = L$

$q = \frac{M^{-3}L \times LT^{-1} \times L}{ML^{-1}T^{-1}}; R = [m^{\circ}L^{\circ}T^{\circ}]$

As we known Ruynold number is dimensionless

7. A flask contain argon and oxygen in the ratio of 3 : 2 in mass and the mixture is kept at 27°C. The ratio of their average kinetic energy per molecule respectively

(A) 3 : 2 (B) 9 : 4 (C) 2 : 3 (D) 1 ; 1

Ans. **Bonus**

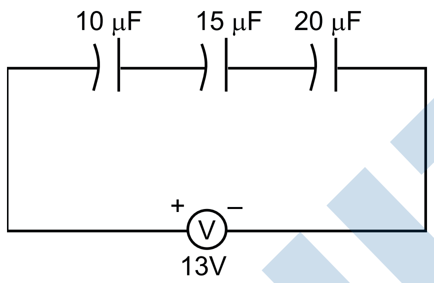
Sol. $\frac{m_1}{m_2} = \frac{3}{2} = f_1 = 3; f_2 = 5$

$T = 27°C; KE_{av} = \frac{f}{2}KT$

$\frac{KE_1}{KE_2} = \frac{3}{5}$

Remark : Ratio of average **translational** kinetic energy per molecule = 1 : 1.

8. The charge on capacitor of capacitance. 15μF in the figure given below is :



(A) 60μC (B) 130μC (C) 260μC (D) 585μC

Ans. **A**

Sol. $\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{15} + \frac{1}{20}$

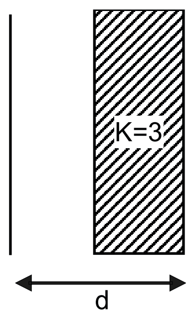
$= \frac{6+4+3}{60}$

q = C_{eq} × V

$\frac{60}{13} \times 13$

q = 60 μC.

9. A parallel plate capacitor with plate area A and plate separation d = 2 m has a capacitance of 4 μF. The new capacitance of the system if half of the space between them is filled with a dielectric material of dielectric constant K = 3 (as shown in figure) will be :



(A) 2μF (B) 32μF (C) 6μF (D) 8μF

Ans. **C**

Sol. $C_1 = \frac{A\epsilon_0}{d/2} = 2C_0\ C_0 = 4$

$C_2 = \frac{A\epsilon_0}{d/2} \times 3 = 6C_0$

$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$

$= \frac{2C_0 \times 6C_0}{8C_0} = \frac{12}{8}C_0$

$= \frac{3}{2} \times 4 = 6.$

10. Sixty four conducting drops each of radius 0.02m and each carrying a charge of 5 μC are combined to form a bigger drop. The ratio of surface density of bigger drop to the smaller drop will be :

(A) 1 : 4 (B) 4 : 1 (C) 1 : 8 (D) 8 : 1

Ans. **B**

Sol. n = 64

r = 0.02 m

q = 5μC

$\frac{\sigma_2}{\sigma_1} = ?$

$nr^3 = R^3$

$R = rn^{1/3}$

$= 4 \times (64)^{1/3}$

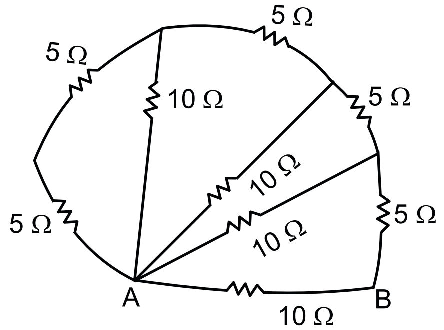
= 4r

$\sigma = \frac{q}{4\pi r^2}$

$\frac{\sigma_2}{\sigma_1} = \frac{q_2}{q_1} \times \left(\frac{r_1}{r_2}\right)^2 = n \times \left(\frac{1}{4}\right)^2$

$= 64 \times \frac{1}{16} = 4.$

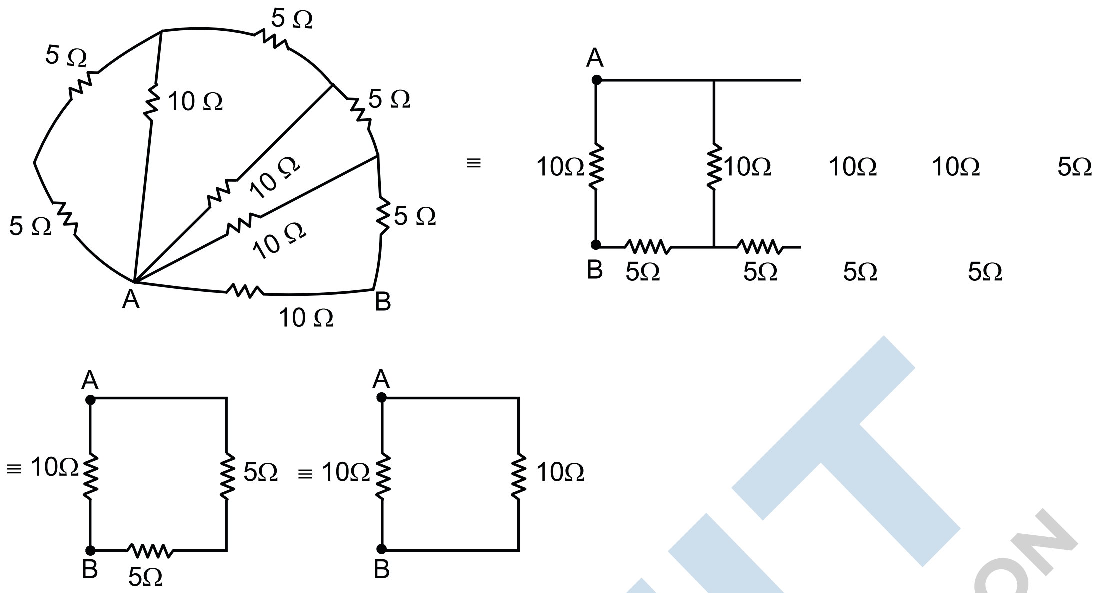
11. The equivalent resistance between points A and B in the given network is :



(A) 65Ω (B) 20Ω (C) 5Ω (D) 65Ω

Ans. C

Sol.



$R_{AB} = 5\Omega$.

12. A bar magnet having a magnetic moment of 2.0×10^5 JT^{-1}, is placed along the direction of uniform magnetic field of magnitude $B = 14 \times 10^{-5}$ T. The work done in rotating the magnet slowly through 60° from the direction of field is :

(A) 14 J (B) 8.4 J (C) 4 J (D) 1.4 J

Ans. A

Sol. $m = 2 \times 10^5$

$B = 14 \times 10^{-5}$ T

$W = U_2 - U_1$

$\theta_1 = 0°$

$\theta_2 = 60°$

$U = -MB\cos\theta$

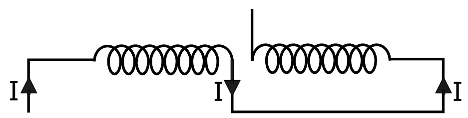
$= -MB\cos\theta_2 + MB\cos\theta_1$

$MB(\cos\theta_1 - \cos\theta_2)$

$$= 2 \times 10^5 \times 14 \times 10^{-5} \times \left(\frac{1}{2} - 1\right)$$

= –14.

13. Two coils of self inductance L_1 and L_2 are connected in series combination having mutual inductance of the coils as M. The equivalent self inductance of the combination will be :



(A) $\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{M}$ (B) $L_1 + L_2 + M$ (C) $L_1 + L_2 + 2M$ (D) $L_1 + L_2 - 2M$

Ans. D

Sol. $L = L_1 + L_2 + M \times 2$ if current in same direction

$L_{eq} = L_1 + L_2 - M \times 2$

14. A metallic conductor of length 1m rotates in a vertical plane parallel to east-west direction about one of its end with angular velocity 5 rad s^{-1}. If the horizontal component of earth's magnetic field is 0.2×10^{-4} T, then emf induced between the two ends of the conductor is "

(A) $5\mu V$ (B) $50\mu V$ (C) 5mV (D) 50mV

Ans. B

Sol. $\ell = 1m$

W = 5

$B_H =$

$\varepsilon = ?$

$$\in = \frac{1}{2}B\omega\ell^2 = \frac{1}{2}\times 0.2\times 10^{-4}\times 5\times 1^2 = 5\times 10^{-5}V = 50\mu V$$

15. Which the correct ascending order of wavelengths ?

(A) $\lambda_{visible} > \lambda_{X-ray} < \lambda_{gamma\text{-}ray} < \lambda_{microwave}$

(B) $\lambda_{gamma\text{-}ray} > \lambda_{X-ray} < \lambda_{visible} < \lambda_{microwave}$

(C) $\lambda_{X-ray} > \lambda_{gamma\text{-}ray} < \lambda_{visible} < \lambda_{microwave}$

(D) $\lambda_{microwave} > \lambda_{visible} < \lambda_{gamma\text{-}ray} < \lambda_{X-ray}$

Ans. B

16. For a specific wavelength 670 nm of light coming from a galaxy moving with velocity ν, the observed wavelength is 670.7 m

(A) $3 \times 10^8\ ms^{-1}$ (B) $3 \times 10^{10}\ ms^{-1}$ (C) $3.13 \times 10^5\ ms^{-1}$ (D) $4.48 \times 10^5\ ms^{-1}$

Ans. C

Sol. $\lambda = 670$ nm

$\Delta\lambda = 0.7$ nm

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{C}; V = \frac{\Delta\lambda}{\lambda}\times C$$

$$= \frac{0.7}{670}\times 3\times 10^8 = 3.13\times 10^5 m/s.$$

17. A metal surface is illuminated by a radiation of wavelength 4500 Å. The ejected photo-electron enters a constant magnetic field of 2 mT making an angle of 90° with the magnetic field. If it starts revolving in a circular path of radius 2 nm, the work function of the metal is approximately :

(A) 1.36 eV (B) 1.69 eV (C) 2.78 eV (D) 2.23 eV

Ans. A

Sol. λ , β, γ, ϕ = ?

$$KE = \frac{nc}{\lambda} - \phi$$

$$r = \frac{mv}{qB} = \frac{\sqrt{2mkE}}{qB}$$

$$KE = \frac{r^2q^2B^2}{2m}$$

$$= \frac{\left(2\times10^{-3}\right)^2 \times 1.6\times10^{-19} \times \left(2\times10^{-3}\right)^2}{2\times9.1\times10^{-31}} eV$$

$$= \frac{10^{-6-9-6+31} \times 2\times1.6\times4}{2\times9.1} eV$$

$$= \frac{2\times6.4}{9.1} eV$$

$$E = \frac{nC}{\lambda} = \frac{12400}{4500} eV$$

$$= \frac{124}{45} eV$$

ϕ = E – KE

$$= \frac{124}{45} - \frac{6.4\times2}{9.1} = 2.755 - 1.4$$

1.355 = 1.36 eV.

18. A radioactive nucleus can decay by two different process Half-life for the first process is 3.0 hours while it is 4.5 hours for the second process. The effective half-life of the nucleus will be :

(A) 3.75 hours (B) 0.56 hours (C) 0.26 hours (D) 1.80 hours

Ans. D

Sol. $T_1 = 3$

$T_2 = 4.5$

$$T_{eq} = \frac{T_1 \times T_2}{T_1 + T_2} = \frac{3\times4.5}{7.5} = 1.8hr.$$

19. The positive feedback is required by an amplifier to act an oscillator. The feedback here means :

(A) External input is necessary to sustain ac signal in output.

(B) A portion of the output is returned back to the input.

(C) Feedback can be achieved by LR network.

(D) The base-collector junction must be forward biased.

Ans. B

20. A sinusoidal wave $y(t) = 40\sin(10 \times 10^6\ \pi t)$ is amplitude modulated by another sinusoidal wav $x(t) = 20\sin(1000\pi t)$. The amplitude of minimum frequency component of modulated signal is :

(A) 0.5 (B) 0.25 (C) 20 (D) 10

Ans. **D**

Sol. $\mu = \frac{Am}{Ac} = \frac{20}{40} = \frac{1}{2}$

Modulated amplitude $= \frac{\mu Ac}{2}$

$= \frac{1}{2} \times \frac{40}{2} = 10.$

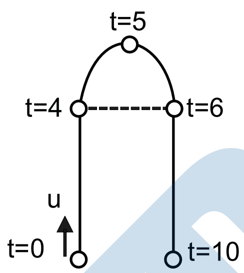
Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

21. A ball is projected vertically upward with an initial velocity of 50 ms^{-1} at t = 0s. At t = 2s, another ball is projected vertically upward with same velocity. At t = ________s. second ball will meet the first ball ($g = 10\ ms^{-2}$).

Ans. **6**

Sol. u = 50



$T = \frac{2 \times 4}{g} = \frac{2 \times 50}{10} = \frac{100}{10} = 10;\ t_a = t_d = 5$

22. A batsman hits back a ball of mass 0.4 kg straight in the direction of the bowler without changing its initial speed of 15 ms^{-1}. The impulse imparted to the ball is __________ Ns.

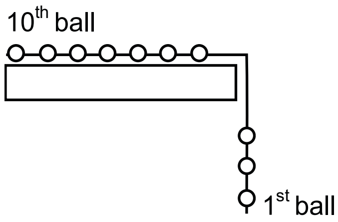
Ans. **12**

Sol. $\Delta p = I = 2mu$

$= 1 \times 0.4 \times 15$

$= 12$

23. A system to 10 balls each of mass 2 kg are connected via massless and unstretchable string. The system is allowed to slip over the edge of a smooth table as shown in figure. Tension on the string between the 7^{th} and 8^{th} ball is ______N when 6^{th} ball just leaves the table.



Ans. **36**

F = ma

6mg = 10 ma

$a = \frac{6g}{10}$

T = ma

$T = 3m \times \frac{6g}{10}$

$= \frac{18}{10} \times 2 \times 10 = 36.$

24. A geyser heats water flowing at a rate of 2.0 kg per minute from 30°C to 70°C. if ______ g min^{-1}

[Heat of combustion = 8 × 10^3 Jg^{-1}, Specific heat of water = 4.2 Jg^{-1} $°C^{-1}$]

Ans. **42**

Sol. $\frac{dm}{dt} = 2kg/min$

$\theta_1 = 30°C$

$\theta_2 = 70°C$

$\frac{\Delta Q}{\Delta t} = \frac{dm}{dt} \times s\Delta\theta$

$\frac{\Delta Q}{\Delta t} = \frac{2}{60} \times 4.2 \times 40 = \frac{m}{t} \times L$

$\frac{\Delta Q}{\Delta t} = \frac{m}{t} \times L$

L = heat of combination

$= 8 \times 10^{3+}$

$\frac{m}{t} = \frac{2 \times 4.2 \times 40}{60 \times 8 \times 10^3}$

$= \frac{4.2}{6} \times 10^{-3} = 0.7 \times 10^{-3} kg/sec. = 0.7g/min. = 42gm/min/ .$

25. A heat engine operates with the cold reservoir at temperature 324 K.

The minimum temperature of the hot reservoir, if the heat engine takes 300 J heat from the hot reservoir and delivers 180 J heat to the cold reservoir per cycle, is ________ K.

Ans. **540**

Sol. $T_2 = 324$

$Q_1 = 300$ J

$Q_2 = 180$ J

$$\frac{T_1}{T_2} = \frac{Q_1}{Q_2}$$

$$T_1 = \frac{300}{180} \times 324 = 540K.$$

26. A set of 20 turning forks is arranged in a series of increasing frequencies. If each fork gives 4 beats with respect to the preceding fork and the frequency of the last fork is twice the frequency of the first, then the frequency of last fork is ________Hz.

Ans. **152**

Sol. n = 20

$f_b = 3$

$f_{last} = 2f_1$

$f_{last} = f_1 + f_b \times (n - 1)$

$2f_1 = f_1 + 4 \times 19$

$f_1 = 76$

$f_{last} = 2 \times 76 = 152.$

27. Two 10 cm long, straight wires, each carrying a current of 5A are kept parallel to each other. If each wire experienced a force of 10^{-5} N, then separation between the wires is ________cm.

Ans. **5**

Sol. ℓ = 10cm

i = 5A

$F = 10^{-5}$ N

d = ?

$$F = \frac{\mu_0 i_1 i_2}{2\pi d} \times \ell$$

$$10^{-5} = \frac{4\pi \times 10^{-7} \times 5^2}{2\pi \times d} \times 0.1$$

$$d = 2 \times \frac{25 \times 0.1 \times 10^{-7}}{10^{-5}}$$

50×10^{-3}m = 5 cm.

28. A small bulb is placed at the bottom of a tank containing water to a depth of $\sqrt{7}$m. The refractive index of water is $\frac{4}{3}$. The area of the surface of water through which light from the bulb can emerge out is $x\pi$ m^2. The value of x is _______.

Ans. 9

Sol. $h = d = \sqrt{7}$

$\mu = \frac{4}{3}$

$A = x\,\pi\,m^2$

$x = ?$

$\sin c = \frac{1}{\mu} = \frac{3}{4}$

$\tan c = \frac{3}{\sqrt{16-9}} = \frac{3}{\sqrt{7}}$

$r = h \tan C$

$A = \pi r^2$

$= \pi h^2 \tan^2 C$

$= \pi \times 7 \times \frac{9}{7} = 9\pi$

$x = 9.$

29. A travelling microscope is used to determine the refractive index of a glass slab. If 40 divisions are there in 1 cm on main scale and 50 Vernier scale divisions are equal to 49 main scale divisions, then least count of the travelling microscope is _______ $\times 10^{-6}$m.

Ans. 5

Sol. $ms = \frac{1}{40}cm$

$50 \times vs = 49 \times ms$

$LC = ms - vs$

$= ms - \frac{49}{50}ms$

$= \frac{1}{50} \times ms$

$= \frac{1}{50} \times \frac{1}{40}cm = \frac{1}{2000} \times 10^{-2}m$

$= 5 \times 10^{-6}$m.

30. The sopping potential for photoelectrons emitted from a surface illuminated by light of wavelength 6630 Å is 0.42 V. If the threshold frequency is $x \times 10^{13}$/s, where x is _________.

(Given, speed light = 3×10^8 m/s, Planck's constant = 6.63×10^{-34}Js)

Ans. 35

Sol. $\lambda_{vs} = v_{th} = ?$

$$e_{vs} = \frac{nc}{\lambda} - h_{v_{th}}$$

$$\frac{e_{vs}}{h} = \frac{c}{\lambda} - v_{th}$$

$$v_{th} = \frac{c}{\lambda} - \frac{ev_s}{h}$$

$$= \frac{3 \times 10^8}{66330 \times 10^{-10}} - \frac{1.6 \times 10^{-19} \times 0.42}{6.63 \times 10^{-34}}$$

$$= \frac{3}{6.630} \times 10^{15} - \frac{1.6 \times 0.42}{6.63} \times 10^{15}$$

$$= 10^{15}\left(\frac{3}{6.630} - \frac{1.6 \times 0.42}{6.63}\right) = 0.4524 - 0.1013$$

$V_{th} = 35.11 \times 10^{13}$.

PART B : CHEMISTRY

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **Only One** is correct.

1. The number of radial and angular nodes in 4d orbitals are, respectively

(A) 1 and 2 (B) 3 and 2 (C) 1 and 0 (D) 2 and 1

Ans. A

Sol. 4d orbitals

Angular nodes = $\ell = 2$

Radial nodes = $(n - \ell - 1) = 4 - 2 - 1 = 1$

2. Match **List I** with **List II.**

List I **Enzyme**	**List II** **Conversion of**
A. Invertase	I. Starch into maltose
B. Zymase	II. Maltose into glucose
C. Diastase	III. Glucose into ethanol
D. Maltase	IV. Cane sugar into glucose

Choose the most appropriate answer from the options given below :

(A) A-III, B-IV, C-II, D-I (B) A-III, B-II, C-I, D-II

(C) A-IV, B-III, C-I, D-II (D) A-IV, B-II, C-III, D-I

Ans. C

3. Which of the following elements is considered as a metalloid ?

(A) Sc (B) Pb (C) Bi (D) Te

Ans. D

Sol. 16^{Th} group : $\underbrace{O \quad S}_{\text{None metal}}$ $\underbrace{Se \quad Te}_{\text{Metalloid}}$ $\underbrace{Po}_{\text{Metal}}$

4. The role of depressants in 'Froth Flotation method' is to

(A) selectively prevent one component of the ore from coming to the froth.

(B) reduce the consumption of oil for froth formation.

(C) stabilize the froth.

(D) enhance non-wettability of the mineral particles.

Ans. A

Sol. In the froth flotation process, the role of the depressants is to separate two sulphide ores by selectively preventing one ore form forming froth.

For example, to separate two sulphide ores, Pbs and ZnS, NaCN is utilized. NaCN forms a combination with zinc on the surface of zinc sulphide, $Na_2[Zn(CN)_4]$, preventing ZnS from foaming selectively.

$4NaCN + ZnS \Rightarrow Na_2[Zn(CN)_4] + Na_2S$

5. Boiling of hard water is helpful in removing the temporary hardness by converting calcium hydrogen carbonate and magnesium hydrogen carbonate to

(A) $CaCO_3$ and $Mg(OH)_2$ (B) $CaCO_3$ and $MgCO_3$

(C) $Ca(OH)_2$ and $MgCO_3$ (D) $Ca(OH)_2$ and $Mg(OH)_2$

Ans. A

Sol. $Mg(HCO_3)_2 \xrightarrow{\text{Boiling}} Mg(OH)_2 + 2CO_2 \uparrow$

$Mg(HCO_3)_2 \xrightarrow{\text{Boiling}} CaCO_3 \downarrow + H_2O + CO_2 \uparrow$

6. s-block element which **cannot** be qualitatively confirmed by the flame test is

(A) Li (B) Na (C) Rb (D) Be

Ans. D

Sol.

Metal	Li	Na	K	Rb	Cs
Colour	Crimson red	Yellow	Violet/Lilac	Red violet	Blue

Metal	Be	Mg	Ca	Sr	Ba
Colour	No colour	No colour	Brick red	Crimson red	Apple green

7. The oxide which contains an odd electron at the nitrogen atom is

(A) N_2O (B) NO_2 (C) N_2O_3 (D) N_2O_5

Ans. B

Sol.

Species	Total e^-
NO_2	23
N_2O	22
N_2O_3	38
N_2O_5	54

8. Which one of the following is an example of disproportionation reaction ?

(A) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

(B) $MnO_4^- + 4H^+ + 4e^- \rightarrow MnO_2 + 2H_2O$

(C) $101^- + 2MnO_4^- + 16H^+ \rightarrow 2Mn^{2+} + 8H_2O + 5I_2$

(D) $8MnO_4^- + 3S_2O_3^{2-} + H_2O \rightarrow 8MnO_2 + 6SO_4^{2-} + 2OH^-$

Ans. A

Sol. $3\overset{+6}{Mn}O_4^{2-} + 4H^+ \longrightarrow 2\overset{+7}{Mn}O_4^- + \overset{+4}{Mn}O_2 + 2H_2O$

9. The most common oxidation state of Lanthanoid elements is + 3, Which of the following is likely to deviate easily from +3. Which of the following is likely to deviate easily from +3 oxidation state ?

(A) Ce(At. No. 58) (B) La(At. No. 57) (C) Lu(At. No. 71) (D) Gd(At. No. 64)

Ans. **A**

Sol. In the lanthanoids, La(III) and Ln(III) compounds the formation of Ce^{IV} is favoured by its noble gas configuration.

10. The measured BOD values for four different water sample (A-D) are as follows :

A = 3 ppm; B=18 ppm; C=21 ppm; D=4 ppm. The water sample which can be called as highly polluted with organic wastes, are

(A) A and B (B) A and D (C) B and C (D) B and D

Ans. **C**

Sol. The sample with BOD value > 14 is considered polluted.

11. The correct order of nucleophilicity is

(A) $F^- > OH^-$ (B) $H_2O > OH^-$ (C) $H_2OH > RO^-$ (D) $NH_2^- > NH_3$

Ans. **D**

Sol. Stronger the base, greater the nucleophilicity.

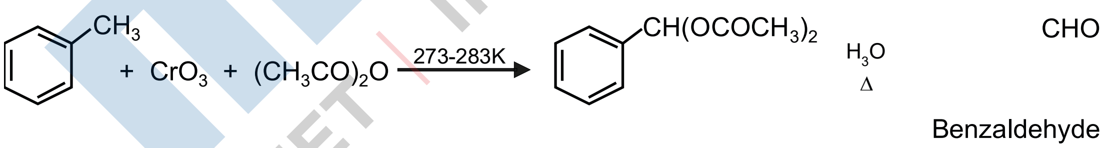
12. Oxidation of toluene to benzaldehyde can be easily carried out with which of the following reagents ?

(A) CrO_3/acetic acid, H_3O^+ (B) CrO_3/acetic anhydride, H_3O^+

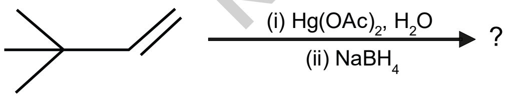
(C) $KMnO_4$/HCl, H_3O^+ (D) CO/HCl, anhydrous $AlCl_3$

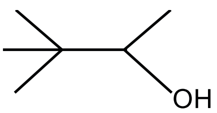
Ans. **B**

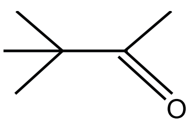
Sol. **Use of chromic oxide (CrO_3):** Toluene or substituted toluene is converted to benzylidene diacetate on treating with chromic oxide in acetic anhydride. The benzylidene diacetate can be hydrolysed to corresponding benzaldehyde with aqueous acid.

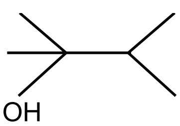


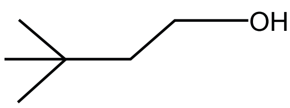
13. The major product in the following reaction



(A) 

(B) 

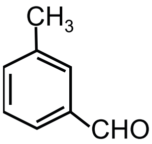
(C) 

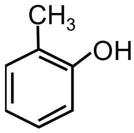
(D) 

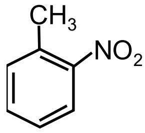
Ans. A

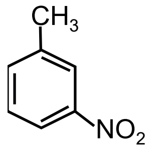
Sol. It is example of oxymercuration-demercuration reaction which proceeds with markownikov addition of water without rearrangement.

14. Halogenation of which one of the following will yield m-substituted product with respect to methyl group as a major product ?

(A) 

(B) 

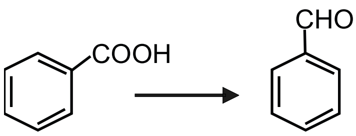
(C) 

(D) 

Ans. C

Sol. OH^- is stronger o, p-directing and direct incoming group to meta position with respect to CH_3 group.

15. The reagent, from the following, which converts benzoic acid to benzaldehyde in one step is



(A) $LiAlH_4$ (B) $KMnO_4$ (C) MnO (D) $NaBH_4$

Ans. C

Sol. It is fact.

16. The final product 'A' in the following reaction sequence

$$CH_3CH_2-\underset{\underset{O}{\|}}{C}-CH_3 \xrightarrow{HCN} ? \xrightarrow[\text{Heat}]{95\%\ H_2SO_4} A$$

(A) $CH_3-CH=\overset{\overset{CH_3}{|}}{C}-COOH$

(B) $CH_3-CH=\underset{\underset{CH_3}{|}}{C}-CN$

(C) $CH_3-CH_2-\overset{\overset{OH}{|}}{\underset{\underset{CH_3}{|}}{C}}-COOH$

(D) $CH_3-CH_2=\underset{\underset{CH_3}{|}}{C}-CONH_2$

Ans. A

Sol. $CH_3CH_2COCH_3 + HCN \rightarrow CH_3CH_2C(CN)OHCH_3 \xrightarrow{95\%H_2SO_4} CH_3CH=C(CN)COOH$

17. Which statement is NOT correct for p-toluenesulphonyl chloride?

(A) It is known as Hinsberg's reagent.

(B) It is used to distinguish primary and secondary amines.

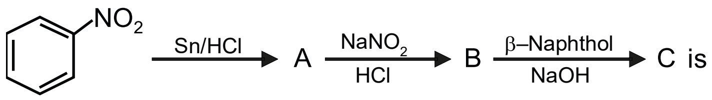
(C) On treatment with secondary amine, it leads to a product, that is soluble alkali.

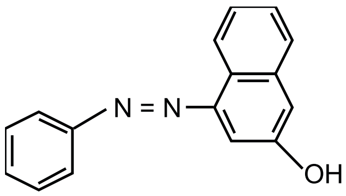
(D) It doesn't react with tertiary amines.

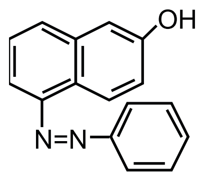
Ans. C

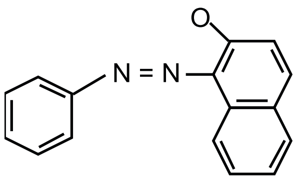
Sol. Secondary amine gives product which is insoluble in KOH.

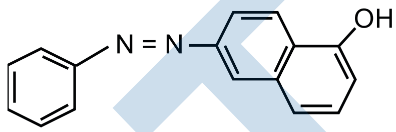
18. The final product 'C' in the following series of reactions



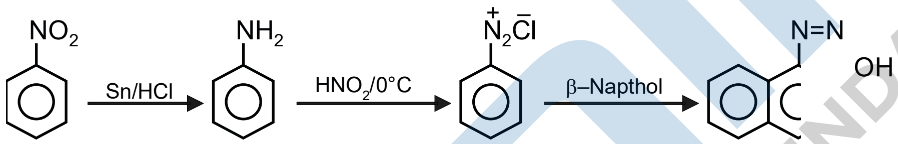
(A) 

(B) 

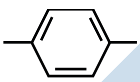
(C) 

(D) 

Ans. **C**

Sol. 

19. Which of the following is NOT an example of synthetic detergent ?

(A) $CH_3-(CH_2)_{11}-$  $-SO_3^-Na^+$

(B) $CH_3-(CH_2)_{16}-COO^-\ Na^+$

(C) $\left[CH_3-(CH_2)_{15}-\overset{\displaystyle CH_3}{\underset{\displaystyle CH_3}{N}}-CH_3\right]^+ Br^-$

(D) $CH_3(CH_2)_{16}COO(CH_2\ CH_2O)_n\ CH_2CH_2\ OH$

Ans. **B**

Sol. Sulphonates and quaternary ammonium salt of long chain fatty acids are used as synthetic detergent option (2) is just a hydrocarbon chain which is insoluble in water.

20. Which one of the following is a water soluble vitamin, that is not excreted easily ?

(A) Vitamin B_2 (B) Vitamin B_1 (C) Vitamin B_6 (D) Vitamin B_{12}

Ans. **D**

Sol. Water soluble vitamins are almost not stored in the body except vitamin B_{12} (cyanocobalamine).

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

21. CNG is an important transportation fuel. When 100 g CNG is mixed with 208 g oxygen in vehicles, it leads to the formation of CO_2 and H_2O and produce large produced in grams is ___.

[Assume CNG to be methane]

Ans. **143**

Sol. $CH_4 + 2O_2(g) \rightarrow CO_2(g) + H_2O$

$\frac{100}{16}$mole $\quad\quad \frac{208}{32}$mole

6.25 $\quad\quad$ 6.5

(LR) $\quad\quad \frac{6.5}{2}$mole

$$W_{CO_2} = \frac{6.5}{2} \times 44 = 143g$$

22. In a solid AB, A atoms are in ccp arrangement and B atoms occupy all the octahedral sites, If two atoms from the opposite faces are removed, then the resultant stoichiometry of the compound is A_xB_y. The value of x is _____.

Ans. **3**

Sol. $A = 8(\text{corner})\frac{1}{8} + 4[\text{Face centre}]\frac{1}{2} = 1 + 2 = 3$

B = 4[in all OV]

Formula = $A_3B_4 = A_xB_y$

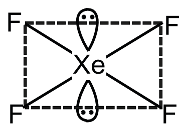
x = 3

23. Amongst SF_4, XeF_4, CF_4 and H_2O, the number of species with two lone pairs of electrons is ____.

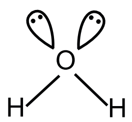
Ans. **NTA answer is (3), Zigyan answer is (2).**

Sol. Compound

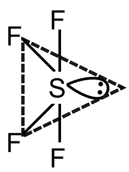
(i) XeF_4

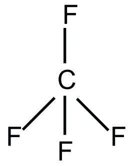


(ii) H_2O



(iii) SF_4



(iv) CF_4 

24. A fish swimming in water body when taken out from the water body is covered with a film of water of weight 36 g. When it is subjected to cooking at 100°C, then the internal energy for vaporization in kJ mol^{-1} is ___________.

[Assume steam to be an ideal gas. Given $\Delta_{vap}H^{\ominus}$ for water at 373 K and 1 bar is 41.1 kJ mol^{-1} : R = 8.31 J $K^{-1}mol^{-1}$]

Ans. **38**

Sol. Weight of water on fish = 36 gram

$$H_2O(\ell)\xrightarrow[P=1bar]{T=100^{\circ}C}H_2O(g)\ ;\ \Delta H_{vap}=41.1\,kJmole^{-1}$$

$\left(\frac{36}{18}\right)=2mole \qquad 2\ mole$

$\Delta H = \Delta U = \Delta n_g RT$

$$(41.1)\times 2=\Delta U+\left(\frac{2\times 8.134}{1000}\right)373$$

$\Delta U = 82.2 - 6.2 = 76$ kJ

so change in internal energy (in kJ/mol) $=\frac{76}{2}=38kJ/mole$

25. The osmotic pressure exerted by a solution prepared by dissolving 2.0 g of protein of molar mass 60 kg mol^{-1} in 200 mL of water at 27° C is _________ Pa.

(use R = 0.083 L bar mol^{-1} K^{-1})

Ans. **415**

Sol. $TT=CRT=\left[\frac{2\times 10^{3}}{60\times 10^{3}\times 200}\right]\times 8.31\times 300=0.415bar=415P_a$

26. 40% of HI undergoes decomposition to H_2 and I_2 at 300 K. $\Delta G^{\ominus}$ for this decomposition reaction at one atmosphere pressure is _______ J mol^{-1}.

(Use R = 8.31 J K^{-1} mol^{-1}; log 2 = 0.3010, In 10 = 2.3, log 3 = 0.477)

Ans. **2735**

Sol. $$HI(g) \rightleftharpoons \frac{1}{2}H_2(g)+\frac{1}{2}I_2(g)$$

Initial mole	1	0	0
mole at eq.	(1 – 0.4)	0.2	0.2

$$K_p=\frac{(0.2)^{1/2}\times(0.2)^{1/2}}{(0.6)}$$

$K_p = \frac{(0.2)}{(0.6)} = \frac{1}{3}$

$\Delta G° = -2.3\ RT \log K_p$

$= -2.3 \times 8.31 \times 300 \log \frac{1}{3}$

$= 2.3 \times 8.31 \times 300 \log 3$

= 2735.07 J/Mole

27. $Cu(s) + Sn^{2+}\ (0.001M) \rightarrow Cu^{2+}\ (0.01M) + Sn(s)$

The Gibbs free energy change for the above reaction at 298 K is x × 10^{-1} kJ mol^{-1}. The value of x is _____.

$\left[\text{Given}: E^1_{Cu^{2+}|Cu} = 0.34V; E^1_{Sn^{2+}|Sn} = -0.14V; F = 96500C\ mol^{-1}\right]$

Ans. **983**

Sol. $E^o_{cell} = E^o_{Su^{2+}|Sn} - E^o_{Cu^{2+}|Cu} = -0.14 - 0.34 = -0.48$

$E_{cell} = E^o_{cell} - \frac{0.059}{2} \log \frac{[Cu^{2+}]}{[Sn^{2+}]}$

$= -0.48 - 0.0295\left[\log \frac{10^{-2}}{10^{-3}}\right]$

= –0.48 – 0.0295

= –0.5095

$\Delta G = -nFE_{cell}$

= –2 × 96500 (–0.5095)

= 98333.5 = 98.33 kJ = 983.3 × 10^{-1} kJ

28. Catalyst A reduces the activation energy for a reaction by 10 kJ mol^{-1} at 300 K. The ratio of rate constants, $\frac{k_T,\text{Catalyed}}{k_T,\text{Uncatalysed}}$ is e^x. The value of x is ____.

[Assume that the pre-exponential factor is same in both the cases. Given R = 8.31 JK^{-1} mol^{-1}]

Ans. **4**

Sol. $K = Ae^{-\frac{Ea}{RT}}$

$K_{cat} = Ae^{-\frac{Ea'}{RT}}$

$K_{absence} = Ae^{-\frac{Ea}{RT}}$

$\frac{K_{catalyst}}{K_{absence}} = e^{\frac{Ea - E_a'}{RT}} = e^{\frac{10 \times 1000}{8.134 \times 300}} = e^4$

29. Reaction of $[Co(H_2O)_6]^{2+}$ with excess ammonia and in the presence of oxygen results into a diamagnetic product. Number of electrons present in t_{2g}-orbitals of the product is _____.

Ans. **6**

Sol. $[Co(H_2O)_6]^{3+} + NH_3(excess) \rightarrow [Co(NH_3)_6]^{3+}(aq) + 6H_2O$

$$\left[Co(NH_3)_6\right]^{3+} \Rightarrow Co^{3+} = 3d^6 4s^0 \Rightarrow t_{2g}^{2,2,2}, e_g^{0,0}$$

30. The moles of methane required to produce 81 g of water after complete combustion is _____ × 10^{-2} mol.

Ans. **225**

Sol. $CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O$

$\frac{1}{2}\left[\frac{81}{18}\right]$mole $\qquad\qquad \left[\frac{81}{18}\right]$mole

= 2.25 mole

225×10^{-2}

PART C : MATHEMATICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **Only One** is correct.

1. Let f : R → R be defined as f(x) = x – 1 and g : R – {1, –1} → R be defined as $g(x) = \frac{x^2}{x^2 - 1}$. Then the function fog is :

(A) one – one but not into function
(B) onto but not one-one function
(C) both one-one and onto function
(D) neither one-one nor onto function

Ans. **D**

Sol. $f(x) = x - 1; g(x) = \frac{x^2}{x^2 - 1}$

f(g(x)) = g(x) – 1

$= \frac{x^2}{x^2 - 1} - 1 = \frac{x^2 - x^2 + 1}{x^2 - 1}$

$f(g(x)) = \frac{1}{x^2 - 1}; x \neq \pm$, even function

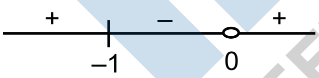
→ Hence f(g(x)) is many one function

$y = \frac{1}{x^2 - 1}$

$y \cdot x^2 - y = 1$

$x^2 = \left(\frac{1+y}{y}\right)$

$\left(\frac{1+y}{y}\right) \geq 0$



Range :– y ∈ (–∞, –1] ∪ (0, ∞)

Hence, Range ≠ Co-domain ⇒ f(g(x)) is into function

2. If the system of equations αx + y + z = 5, x + 2y + 3z = 4, x + 3y + 5z = β, has infinitely many solutions, then the ordered pair (α, β) is equal to :

(A) (1, –3)
(B) (–1, 3)
(C) (1, 3)
(D) (–1, –3)

Ans. **C**

For infinitely many solutions.

$\Delta = 0 = \Delta_x = \Delta_y = \Delta_z$

Sol. $\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix} = 0$

$\Rightarrow \alpha(10 - 9) - 1(5 - 3) + 1(3 - 2) = 0$

$\Rightarrow \alpha - 2 + 1 = 0$

$\Rightarrow \alpha = 1$

$\Delta_x = \begin{vmatrix} 5 & 1 & 1 \\ 4 & 2 & 3 \\ \beta & 3 & 5 \end{vmatrix} = 0$

$\Rightarrow 5(10 - 9) - 1(20 - 3\beta) + 1(12 - 2\beta)$

$\Rightarrow 5 - 20 + 3\beta + 12 - 2\beta$

$\Rightarrow -3 + \beta = 0$

$\Rightarrow \beta = 3$

3. if $\sum_{n=1}^{\infty} \frac{1}{\left(3+(-1)^n\right)^n}$ and $B = \sum_{n=1}^{\infty} \frac{(-1)^n}{\left(3+(-1)^n\right)^n}$, then $\frac{A}{B}$ is equal to

(A) $\frac{11}{9}$ (B) 1 (C) $-\frac{11}{9}$ (D) $-\frac{11}{3}$

Ans. C

Sol. $A = \left(\frac{1}{2} + \frac{1}{4^2} + \frac{1}{2^3} + \frac{1}{4^4} + \ldots\ldots\infty\right)$

$A = \left(\frac{1}{2} + \frac{1}{2^3} + \ldots\ldots\infty\right) + \left(\frac{1}{4^2} + \frac{1}{4^4} + \ldots\ldots\infty\right)$

$A = \left(\frac{\frac{1}{2}}{1-\frac{1}{4}} + \frac{\frac{1}{16}}{1-\frac{1}{16}}\right)$

$\Rightarrow A = \frac{1}{2} \times \frac{4}{3} + \frac{1}{16} \times \frac{16}{15} \Rightarrow A = \frac{11}{15}$

$B = \frac{-1}{2} + \frac{1}{4^2} + \frac{-1}{2^3} + \frac{1}{4^4} + \ldots\ldots\infty$

$B = \left(\frac{-1}{2} + \frac{-1}{2^3} + \ldots\infty\right) + \left(\frac{1}{4^2} + \frac{1}{4^4} + \ldots\ldots\infty\right)$

$B = \frac{-\frac{1}{2}}{1-\frac{1}{4}} + \frac{\frac{1}{16}}{1-\frac{1}{16}}$

$\Rightarrow B = -\frac{1}{2} \times \frac{4}{3} + \frac{1}{16} \times \frac{16}{15}$

$B = -\frac{9}{15}$

$\frac{A}{B} = \frac{11}{15} \times \frac{15}{(-9)}$

$\frac{A}{B} = -\frac{11}{9}$

4. $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4}$ is equal to :

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{6}$ (D) $\frac{1}{12}$

Ans. C

Sol. $\lim_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4}; \left(\frac{0}{0}\right)$

$$\lim_{x\to 0} \left(\frac{2 \cdot \sin\left(\frac{x+\sin x}{2}\right) \cdot \sin\left(\frac{x-\sin x}{2}\right)}{x^4} \right)$$

$$\lim_{x\to 0} 2 \left(\frac{\sin\left(\frac{x+\sin x}{2}\right)}{\left(\frac{x+\sin x}{2}\right)} \right) \left(\frac{\sin\left(\frac{x-\sin x}{2}\right)}{\left(\frac{x-\sin x}{2}\right)} \right) \left(\frac{\left(\frac{x+\sin x}{2}\right)}{x^4} \right) \left(\frac{x-\sin x}{2} \right)$$

$$\lim_{x\to 0} \left(\frac{x^2 - \sin^2 x}{2x^4} \right) : \left(\frac{0}{0}\right)$$

Apply L–Hospital Rule :

$$\lim_{x\to 0} \frac{2x - 2\sin x \cos x}{2.4x^3}$$

$$\lim_{x\to 0} \frac{2x - \sin 2x}{8x^3}; \frac{0}{0}$$: Again apply L–Hospital rule

$$\lim_{x\to 0} \frac{2 - 2\cos(2x)}{8(3)x^2}$$

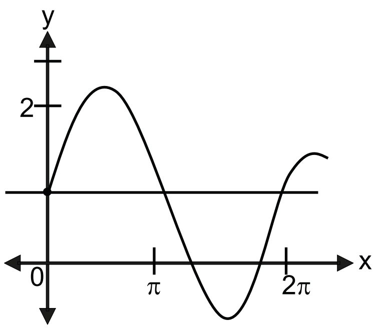
$$\lim_{x\to 0} \frac{2(1 - \cos(2x))}{24(4x^2)} \times 4 \Rightarrow \frac{2}{24} \times \frac{1}{2} \times 4 \Rightarrow \frac{1}{6}$$

5. Let f(x) = min {1, 1 + x sin x}, $0 \le x \le 2\pi$. If m is the number of points, where f is not differentiable an n is the number of points, where f is not continuous, then the ordered pair (m, n) is equal to

(A) (2, 0) (B) (1, 0) (C) (1, 1) (D) (2, 1)

Ans. B

Sol.



No. of non-differentiable points = 1 (m)

No. of not continuous points = 0 (n)

(m, n) = (1, 0)

6. Consider a cuboid of sides 2x, 4x and 5x and a closed hemisphere of radius r. If the sum of their surface areas is a constant k, then the ratio x : r, for which the sum of their volumes is maximum is:

(A) 2 : 5 (B) 19 : 45 (C) 3 : 8 (D) 19 : 15

Ans. **B**

Sol. Surface area = $76x^2 + 3\pi r^2$ = constant (K)

$$V = 40x^3 + \frac{2}{3}\pi r^3$$

$[76x^2 + 3\pi r^2 = K]$

$$r^2 = \frac{K - 76x^2}{3\pi}$$

$$r = \left(\frac{K - 76x^2}{3\pi}\right)^{\frac{1}{2}}$$

$$V = 40x^3 + \frac{2}{3}\pi\left(\frac{K - 76x^2}{3\pi}\right)^{\frac{3}{2}}$$

$$\frac{dV}{dx} = 120x^3 + \frac{2}{3}\pi.\frac{3}{2}\left(\frac{K - 76x^2}{3\pi}\right)^{\frac{1}{2}}.\left(\frac{-76(2x)}{3\pi}\right)$$

Put

$$\frac{dV}{dx} = 0 \Rightarrow 120x^2 + \frac{2}{3}\pi.\frac{3}{2}\left(\frac{K - 76x^2}{3\pi}\right)^{\frac{1}{2}}.\left(\frac{-76(2x)}{3\pi}\right) = 0$$

$$\Rightarrow 120x^2 = \frac{152x}{3}\left(\frac{k - 76x^2}{3\pi}\right)^{\frac{1}{2}}$$

$$\Rightarrow \frac{45}{19}x^2 = x\left(\frac{k - 76x^2}{3\pi}\right)^{\frac{1}{2}}; x \neq 0$$

$$\Rightarrow \frac{45}{19}x = \left(\frac{k-76x^2}{3\pi}\right)^{\frac{1}{2}} \Rightarrow \left(\frac{45}{19}\right)^2 x^2 = \frac{k-76x^2}{3\pi}$$

$$\Rightarrow \left(\frac{45}{19}\right)^2 x^2 = r^2 \Rightarrow \frac{x^2}{r^2} = \left(\frac{19}{45}\right)^2$$

$$\Rightarrow \frac{x}{r} = \frac{19}{45}$$

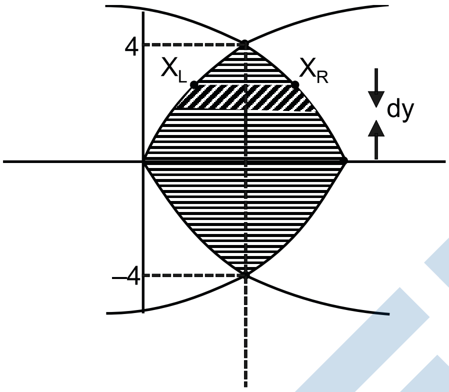
7. The area of the region bounded by $y^2 = 8x$ and $y^2 = 16\,(3 - x)$ is equal to :

(A) $\frac{32}{3}$ (B) $\frac{40}{3}$ (C) 16 (D) 19

Ans. C

Sol. $y^2 = 8x$; $y^2 = 16(3 - x)$

$y^2 = -16(x - 3)$



Finding their intersection pts.

$y^2 = 8x$ & $y^2 = -16\,(x - 3)$

$8x = -16x + 48$

$24x = 48$

$x = 2;\ y = \pm 4$

$$A = 2.\int_0^4 (x_R - x_L)\,dy$$

Required Area

$$= 2.\int_0^4 \left(\underbrace{3 - \frac{y^2}{16}}_{(x_R} - \underbrace{\frac{y^2}{8}}_{x_L)}\right) dy$$

$$= 2\left(3y - \frac{y^3}{3\times 16} - \frac{y^3}{3\times 8}\right)_0^4$$

$$= 2\left(3\times 4 - \frac{4\times 4\times 4}{3\times 16} - \frac{4\times 4\times 4\times 2}{3\times 8\times 2}\right)$$

$$= 2\left(12 - \frac{4}{3} - \frac{8}{3}\right) = 2 \times 12\left(1 - \frac{1}{3}\right) = 2 \times 12 \times \frac{2}{3} = 16$$

8. If $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x) + c, g(1) = 0$, then $g\left(\frac{1}{2}\right)$ is equal to :

(A) $\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) + \frac{\pi}{3}$

(B) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) + \frac{\pi}{3}$

(C) $\log_e\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) - \frac{\pi}{3}$

(D) $\frac{1}{2}\log_e\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right) - \frac{\pi}{6}$

Ans. A

Sol. $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}dx = g(x) + c$

Put x = cos 2θ

dx = –2sin2θ · dθ

$$\int \frac{1}{\cos 2\theta}\tan\theta(-4\sin\theta \cdot \cos\theta)d\theta$$

$$= \int \frac{1}{\cos 2\theta}(-4\sin^2)d\theta$$

$$= -2\int \frac{1-\cos 2\theta}{\cos 2\theta}d\theta$$

$$= -\frac{2}{2}\text{In}|\sec 2\theta + \tan 2\theta| + 2\theta + c$$

= In|sec2θ – tan2θ| + 2θ + c

$$= \text{In}\left|\frac{1-\sin 2\theta}{\cos 2\theta}\right| + \cos^{-1}x + c$$

$$= \underbrace{\text{In}\left|\frac{1-\sqrt{1-x^2}}{x}\right| + \cos^{-1}x}_{g(x)} + c$$

∴ g(1) = 0

$$g(x) = \text{In}\left|\frac{1-\sqrt{1-x^2}}{x}\right| + \cos^{-1}x$$

$$g\left(\frac{1}{2}\right) = \text{In}|2-\sqrt{3}| + \frac{\pi}{3}$$

$$g\left(\frac{1}{2}\right) = \text{In}\left|\frac{\sqrt{3}-1}{\sqrt{3}+1}\right| + \frac{\pi}{3}$$

9. If y = y(x) is the solution of the differentiable equation $x\frac{dy}{dx}+2y=xe^x, y(1)=0$ then the local maximum value of the function z(x) = $x^2y(x)-e^x$, $x \in R$ is :

(A) 1 – e (B) 0 (C) $\frac{1}{2}$ (D) $\frac{4}{e}-e$

Ans. D

Sol. $x\frac{dy}{dx}+2y=xe^x$

$\frac{dy}{dx}+\frac{2y}{x}=e^x$

I.F. = x^2

$y.x^2 = \int x^2e^x\,dx$

$= \int e^x\,(x^2 + 2x - 2x - 2 + 2)dx$

$yx^2 = e^x\,(x^2 - 2x + 2) + c$

y(1) = 0

0 = e(1 + 0) + c

c = –e

$z(x) = x^2y(x) - e^x$

$= e^x\,(x^2 - 2x + 2) - e - e^x$

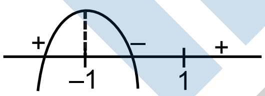
$= e^x\,(x - 1)^2 - e$

$\frac{dz}{dx}=e^x.2(x-1)+e^x(x-1)^2=0$

$x^x\,(x - 1)\,(2 + x - 1) = 0$

$e^x\,(x - 1)\,(x + 1) = 0$

x = –1, 1



x = –1 local maxima. Then maximum value is

$z(-1)=\frac{4}{3}-e$

10. If the solution of the differentiable equation $\frac{dy}{dx}+e^x\left(x^2-2\right)y=\left(x^2-2x\right)\left(x^2-2\right)e^{2x}$ satisfies y(0) = 0, then the value of y(2) is _______.

(A) –1 (B) 1 (C) 0 (D) e

Ans C

Sol. $\text{I.F.}=e^{\int e^x\left(x^2-2\right)dx}=e^{\int e^x\left(x^2-2x+2x-2\right)}dx$

$= e^{e^{x(x^2-2x)}}$

$y.e^{e^{x(x^2-2x)}} = \int e^{e^{x(x^2-2x)}} e^x(x^2-2x)(x^2-2)e^x dx$

Let $e^x(x^2 - 2x) = t$

$y.e^{e^{x(x^2-2x)}} = \int e^t.t\,dt$

So,

At x = 0, t = 0

x = 2, t = 0

$= t\cdot e^t - e^t + c$

$x = 0 ; 0 \cdot 1 = 0 - 1 + c \Rightarrow c = 1$

for $x = 2; y \cdot 1 = 0 - 1 + 1 = 0$

$y(2) = 0$

11. If m is the slope of a common tangent to the curves $\frac{x^2}{16}+\frac{y^2}{9}=1$ and $x^2 + y^2 = 12$, then $12m^2$ is equal to

(A) 6 (B) 9 (C) 10 (D) 12

Ans **B**

Sol. $\frac{x^2}{16}+\frac{y^2}{9}=1$

equation of tangent to the ellipse is

$y = mx \pm \sqrt{a^2m^2 + b^2}$

$y = mx \pm \sqrt{16m^2 + 9}$ ......(i)

$x^2 + y^2 = 12$

equation of tangent to the circle is

$y = mx \pm \sqrt{12}\sqrt{1+m^2}$ ....(ii)

for common tangent equate eq.(i) and (ii)

$\Rightarrow 16m^2 + 9 = 12(1 + m^2)$

$16m^2 - 12\,m^2 = 3$

$4m^2 = 3$

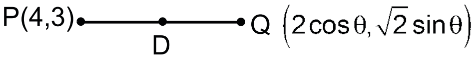
$12m^2 = 9$

12. The locus of the mid point of the line segment joining the point (4, 3) and the points on the ellipse $x^2 + 2y^2 = 4$ is an ellipse with eccentricity :

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

Ans C

Sol. $\frac{x^2}{4}+\frac{y^2}{2}=1$



Coordinate of D is

$$\left(\frac{2\cos\theta+4}{2}, \frac{\sqrt{2}\sin\theta+3}{2}\right)=(h,k)$$

$$\frac{2h-4}{2}=\cos\theta \qquad \ldots(i)$$

$$\frac{2k-3}{\sqrt{2}}=\sin\theta \qquad \ldots.(ii)$$

$(i)^2 + (ii)^2$, then we get

$$\left(\frac{2h-4}{2}\right)^2+\left(\frac{2k-3}{\sqrt{2}}\right)^2=1 \Rightarrow \frac{(x-2)^2}{1}+\frac{\left(y-\frac{3}{2}\right)^2}{\left(\frac{1}{2}\right)}=1$$

$\therefore$ Required eccentricity is

$$e=\sqrt{1-\frac{1}{2}}\frac{1}{\sqrt{2}}$$

13. The normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{9}=1$ at the point $\left(8,3\sqrt{3}\right)$ on it passes through the point :

(A) $\left(15,-2\sqrt{3}\right)$ (B) $\left(9,2\sqrt{3}\right)$ (C) $\left(-1,9\sqrt{3}\right)$ (D) $\left(-1,6\sqrt{3}\right)$

Ans. C

Sol. $\frac{x^2}{a^2}-\frac{y^2}{9}=1 : \left(8,3\sqrt{3}\right)$ lie on Hyperbola then

$$\frac{64}{a^2}-\frac{27}{9}=1 \Rightarrow a^2=\frac{64}{4}=16$$

equation of normal at $\left(8,3\sqrt{3}\right)$:

$$\frac{16x}{8}+\frac{9y}{3\sqrt{3}}=16+9$$

$$2x+\sqrt{3}y=25$$

Check options.

14. If the plane 2x + y – 5z = 0 is rotated about its line of intersection with the plane 3x – y + 4z – 7 = 0 by an angle of $\frac{\pi}{2}$, then the plane after the rotation passes through the point :

(A) (2, –2, 0) (B) (–2, 2, 0) (C) (1, 0, 2) (D) (–1, 0, –2)

Ans. C

Sol. $(2x + y - 5z) + \lambda(3x - y + 4z - 7) = 0$

Rotated by $\pi/2$

$(2 + 3\lambda)x + (1 - \lambda)y + (-5 + 4\lambda)z - 7\lambda = 0$

$2x + y - 5z = 0$

$2(2 + 3\lambda) + (1 - \lambda) - 5(-5 + 4\lambda) = 0$

$\Rightarrow 4 + 6\lambda + 1 - \lambda + 25 - 20\lambda = 0$

$30 = 15\lambda$

$\lambda = 2$

Required plane :– $8x - y + 3z - 14 = 0$

Check options

15. If the lines $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{j} - \hat{k})$ and $\vec{r} = (\alpha\hat{i} - \hat{j}) + \mu(2\hat{j} - 3\hat{k})$ are co-planar, then distance of the plane containing these two lines from the point (•, 0, 0) is :

(A) $\frac{2}{9}$ (B) $\frac{2}{11}$ (C) $\frac{4}{11}$ (D) 2

Ans. B

Sol. $\vec{r} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(3\hat{j} - \hat{k})$ .....L_1

$\vec{r} = (\alpha\hat{i} - \hat{j}) + \mu(2\hat{i} - 3\hat{k})$ .....L_2

• L1 and L2 are coplanar

$$\therefore \begin{vmatrix} 0 & 3 & -1 \\ 2 & 0 & -3 \\ (1-\alpha) & 0 & 1 \end{vmatrix} = 0$$

–3 (2 + 3 (1 – •)) = 0

2 + 3 – 3• = 0

• 3• = 5

$$\Rightarrow \alpha = \frac{5}{3}$$

Now,

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & -1 \\ 2 & 0 & -3 \end{vmatrix} = \hat{i}(-9) - \hat{j}(2) + k(-6)$$

= (9,. 2, 6)

Equation of plane :

$9(x - 1) + 2(y + 1) + 6(z - 1) = 0$

$9x + 2y + 6z - 13 = 0$

Perpendicular distance from(•, 0, 0)

$$= \left| \frac{\left(9 \cdot \frac{5}{3} + 0 + 0 - 13\right)}{\sqrt{18+36+4}} \right| = \frac{2}{\sqrt{121}} = \frac{2}{11}$$

16. Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}, \vec{b} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$ be three given vectors. Let $\vec{v}$ be a vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{2}{\sqrt{3}}$. If $\vec{v}.\hat{j} = 7$, then $\vec{v}.(\hat{i} + \hat{k})$ is equal to :

(A) 6 (B) 7 (C) 8 (D) 9

Ans. D

Sol. $\vec{v} = \lambda\vec{a} + \mu\vec{b}$

$\vec{v} = \lambda(1,1,2) + \mu(2,-3,1)$

$\vec{v} = (\lambda + 2\mu\lambda - 3\mu, 2\lambda + \mu)$

$\vec{v}.\hat{j} = 7$

$\vec{v}.\frac{\vec{c}}{|\vec{c}|} = \frac{2}{\sqrt{3}}$

$\lambda - 3\mu = 7$

$\vec{v}.\vec{c} = 2$

$\lambda + 2\mu - \lambda + 3\mu + 2\lambda + \mu = 2$

$2\lambda + 6\mu = 2$

$\lambda + 3\mu = 1$

$\lambda - 3\mu = 7$

$2\lambda = 8$

$\lambda = 4$

$\mu = -1$

We get $\vec{v} = (2,7,7)$

17. The mean and standard deviation of 50 observations are 15 and 2 respectively. If was found that one incorrect observation was taken such that the sum of correct and incorrect observations is 70. If the correct mean is 16, then the correct variance is equal to :

(A) 10 (B) 36 (C) 43 (D) 60

Ans. C

Sol. No. of observations:– 50

mean$(\bar{x}) = 15$

Standard deviation (σ) = 2

Let incorrect observations is x_1 & correct observation is (x'_1)

Given $x_1 + x'_1 = 70$

$$\bar{x} = \frac{x_1 + x_2 + ...x_{50}}{50} = 15(\text{given})$$

$\Rightarrow x_1 + x_2 + ...x_{50} = 750$ ....(i)

Now

Mean of correct observation is 16

$$\frac{x'_1 + x_2 + ...x_{50}}{50} = 16$$

$x'_1 + x_2 + x_3 + ...x_{50} = 16 \times 50$ ....(ii)

eq. (ii) – eq. (i)

$\Rightarrow x'_1 - x_1 = 16 \times 50 - 15 \times 50$

$x'_1 - x_1 = 50$ & $x_1 - x'_1 = 70$

$x'_1 = 60$

$x_1 = 10$

$$\Rightarrow 4 = \frac{x_1^2 + x_2^2 + ... + x_{50}^2}{50} - 15^2 \quad ....(iii)$$

$$\Rightarrow \sigma^2 = \frac{x_1'^2 + x_2^2 + ... + x_{50}^2}{50} - 16^2 \quad ....(iv)$$

from (iii)

$$\Rightarrow 4 = \frac{(10)^2}{50} + \frac{x_2^2 + x_3^2 + ... + x_{50}^2}{50} - 225$$

$$\Rightarrow 4 = 2 - 225 + \frac{(x_2^2 + x_3^2 + ... + x_{50}^2)}{50}$$

$$\Rightarrow 227 = \frac{(x_2^2 + x_3^2 + ... + x_{50}^2)}{50}$$

From (iv)

$$\sigma^2 = \frac{(60)^2}{50} + \left(\frac{x_2^2 + x_3^2 + ... + x_{50}^2}{50}\right) - (16)^2$$

$$\sigma^2 = \frac{60 \times 60}{50} + 227 - 256$$

$\sigma^2 = 72 + 227 - 256$

$\sigma^2 = 43$

18. 16sin(20°) sin(40°) sin(80°) is equal to :

(A) $\sqrt{3}$ (B) $2\sqrt{3}$ (C) 3 (D) $4\sqrt{3}$

Ans. B

Sol. 16 sin20° sin40° sin80°

= 16 sin40° sin20° sin80°

= 4(4 sin (60 – 20) sin (20) sin (60 + 20))

= 4 × sin (3 × 20°)

[∵ sin 3θ = 4 sin(60 – θ) × sinθ × sin (60 + θ)]

= 4 × sin 60°

$= 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$

19. If the inverse trigonometric function take principle values, then

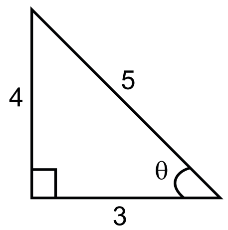
$\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\left(\frac{4}{3}\right)\right) + \frac{2}{5}\sin\left(\tan^{-1}\left(\frac{4}{3}\right)\right)\right)$ is equal to :

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Ans. C

Sol. Let

$\tan^{-1}\frac{4}{3} = \theta \Rightarrow \tan\theta = \frac{4}{3}$



$E = \cos^{-1}\left(\frac{3}{10}\cos\theta + \frac{2}{5}\sin\theta\right)$

$= \cos^{-1}\left(\frac{3}{10} \times \frac{3}{5} + \frac{2}{5} \cdot \frac{4}{5}\right)$

$= \cos^{-1}\left(\frac{9}{50} + \frac{8}{25}\right) = \cos^{-1}\left(\frac{25}{50}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

20. Let r ∈ {p, q, ~p, ~q} be such that the logical statement $r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$ is a tautology. Then 'r' is equal to

(A) p (B) q (C) ~p (D) ~ q

Ans. C

Sol. By options

(1)

p = r	q	~p	r ∨ (~p)	(p ∧ q)	(p ∧ q) ∨ r	r ∨ (~p) ⇒ (p ∧ q) ∨ r
T	F	F	T	F	T	T
F	T	T	T	F	F	F
T	T	F	T	T	T	T

F	F	T	T	F	F	F

(2)

p	~p	$r \vee (\sim p)$	q = r	$(p \wedge q)$	$(p \wedge q) \vee r$	$r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$
T	F	T	T	T	T	T
F	T	T	T	F	T	T
T	F	F	F	F	F	T
F	T	T	F	F	F	F

(3)

p	q	r = ~q	$r \vee (\sim p)$	$(p \wedge q)$	$(p \wedge q) \vee r$	$r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$
T	T	F	F	T	T	T
F	T	T	T	F	T	T
T	F	F	F	F	F	T
F	F	T	T	F	T	T

(4)

~p	p	q	$r \vee (\sim p)$	r = ~q	$(p \wedge q)$	$(p \wedge q) \vee r$	$r \vee (\sim p) \Rightarrow (p \wedge q) \vee r$
F	T	T	F	F	T	T	T
F	T	F	T	T	F	T	T
T	F	T	T	F	F	F	F
T	F	F	T	T	F	T	T

Now final answer is option no. 3

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

21. Let $f : R \to R$ satisfy $f(x + y) = 2^x f(y) + 4^y f(x)$, $\forall x, y \in R$. If $f(2) = 3$, then $14 \cdot \frac{f'(4)}{f'(2)}$ is equal to _____.

Ans. **248**

Sol. Put y = 2

$f(x + y) = 2^x.f(y) + 4^y.f(x).$

$f(x + 2) = 2^x.3 + 16f(x)$

$f'(x + 2) = 16f'(x) + 3.2^x \ln2$

$f'(4) = 16f'(2) + 12\ln2$ ....(i)

$f(y + 2) = 4f(y) + 3.4^y$

$f'(y + 2) = 4f'(y) + 3.4^y \ln4$

$f'(4) = 4f'(2) + 96 \ln2$ ....(ii)

solving eq. (i) and (ii), we get

$f'(2) = 7 \ln2$

from equation (i), we get

$f'(4) = 124 \ln2$

Now, $\Rightarrow 14 \cdot \frac{f'(4)}{f'(2)}$

$14 \times \frac{124\ell n2}{7\ell n2}$

= 248.

22. Let p and q be two real numbers such that p + q = 3 and $p^4 + q^4 = 369$. Then $\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}$ is equal to ____.

Ans. **4**

Sol. $p + q = 3$ $p^4 + q^4 = 369$

$\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}$

$(p + q)^2 = 9$

$p^2 + q^2 = 9 - 2pq$

$\frac{1}{\left(\frac{1}{p}+\frac{1}{q}\right)^2} = \frac{(qp)^2}{(q+p)^2} = \frac{(qp)^2}{9}$

$p^4 + q^4 = (p^2 + q^2)^2 - 2p^2q^2$

$369 = (9 - 2pq)^2 - 2(pq)^2$

$369 = 81 + 4p^2q^2 - 36pq - 2p^2q^2$

$288 = 2p^2q^2 - 36pq$

$144 = p^2q^2 - 18pq$

$(pq)^2 - 2 \times 9 \times pq + 9^2 = 144 + 9^2$

$(pq - 9)^2 = 225$

$pq - 9 = \pm 15$

$pq = \pm 15 + 9$

$pq = 24, -6$

(24 is rejected because $p^2 + q^2 = 9 - 2pq$ is negative)

$\frac{(qp)^2}{9} = \frac{1(-6)^2}{9} = 4$

23. If $z^2 + z + 1 = 0$, $z \in C$, then $\left|\sum_{n=1}^{15}\left(Z^n + (-1)^n \frac{1}{Z^n}\right)^2\right|$ is equal to _____.

Ans. **2**

Sol. $z^2 + z + 1 = 0 \Rightarrow z = w, w^2$

$\left|\sum_{n=1}^{15}\left(z^n + (-1)\frac{1}{z^n}\right)^2\right| = \left|\sum_{n=1}^{15}\left(z^{2n} + \frac{1}{z^{2n}} + 2(-1)^n\right)\right|$

$\left|\sum_{n=1}^{15} w^{2n} + \frac{1}{w^{2n}} + 2(-1)^n\right|$

$\left|\frac{w^2(1-w^{30})}{1-w^2} + \frac{\frac{1}{w^2}\left(1-\frac{1}{w^{30}}\right)}{1-\frac{1}{w^2}} + 2(-1)\right|$

$\left|\frac{w^2(1-1)}{1-w^2} + \frac{\frac{1}{w^2}(1-1)}{1-\frac{1}{w^2}} - 2\right|$

$= |0 + 0 - 2| = 2$

24. Let $X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $Y = \alpha I + \beta X + \gamma X^2$ and $Z = \alpha^2 I - \alpha\beta X + (\beta^2 - \alpha\gamma)X^2$, $\alpha, \beta, \gamma \in R$. If $Y^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$,

then $(\alpha - \beta + \gamma)^2$ is equal to _______.

Ans. **100**

Sol. $X=\begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}, X^2=\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}$

$Y=\begin{bmatrix}\alpha&\beta&\gamma\\0&\alpha&\beta\\0&0&\alpha\end{bmatrix}, Z=\begin{bmatrix}\alpha^2&-\alpha\beta&\beta^2-\alpha\gamma\\0&\alpha^2&-\alpha\beta\\0&0&\alpha^2\end{bmatrix}$

$Y.Y^{-1}=I$

$$\begin{bmatrix}\alpha&\beta&\gamma\\0&\alpha&\beta\\0&0&\alpha\end{bmatrix}\begin{bmatrix}\frac{1}{5}&\frac{-2}{5}&\frac{1}{5}\\0&\frac{1}{5}&\frac{-2}{5}\\0&0&\frac{1}{5}\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$\frac{\alpha}{5}=1\Rightarrow\alpha=5$

$-\frac{2}{5}\alpha+\frac{\beta}{5}=0\Rightarrow\beta=10$

$\frac{\alpha}{5}-\frac{2\beta}{5}+\frac{\gamma}{5}=0\Rightarrow\gamma=15$

$\Rightarrow(\alpha-\beta+\gamma)^2=(5-10+15)^2=100$

25. The total number of 3–digit numbers, whose greatest common divisor with 36 is 2, is ______.

Ans. **150**

Sol. 36 = 2 × 2 × 3 × 3

Number should be odd multiple of 2 and does not having factor 3 and 9 Odd multiple of 2 are 102, 106, 110, 114 ……. 998 (225 no.)

No. of multiple of 3 are

102, 114, 126 ……. 990 (75 no.)

Which are also included multiple of 9

Hence,

Required = 225 – 75 = 150

26. If $\left({}^{40}C_0\right)+\left({}^{41}C_1\right)+\left({}^{42}C_2\right)+......+\left({}^{60}C_{20}\right)=\frac{m}{m}{}^{60}C_{20}$, m and n are coprime, then m + n is equal to ____.

Ans. **102**

Sol. ${}^{40}C_0+{}^{41}C_1+{}^{42}C_2+.........{}^{59}C_{19}+{}^{60}C_{20}$

$\left(\frac{1}{41}+1\right){}^{41}C_1+{}^{42}C_2+......$

$\left[\frac{42}{41}\left(\frac{2}{42}\right)+1\right]{}^{42}C_2+{}^{43}C_3+......$

$\left(\frac{2}{41}+1\right)\,{}^{42}C_2+{}^{43}C_3+\ldots\ldots$

$\left(\frac{43}{41}\times\frac{3}{43}+1\right)\,{}^{43}C_3+{}^{44}C_4+\ldots\ldots$

$\frac{3+41}{41}.\,{}^{43}C_3+\ldots..$

Similarly :

$\frac{20+41}{41}$

$\Rightarrow$ m = 61 ; n = 41

m + n = 102

27. If $a_1(>0)$, a_2, a_3, a_4, a_5 are in a G.P., $a_2 + a_4 = 2a_3 + 1$ and $3a_2 + a_3 = 2a_4$, then $a_2 + a_4 + + 2a_5$ is equal to ______.

Ans. **40**

Sol. $a_1>0$, a_2, a_3, a_4, $a_5 \rightarrow$ G.P.

$3a_2 + a_3 = 2a_4$

$3ar + ar^2 = 2ar^3$

$3 + r = 2r^2$

$2r^2 - r - 3 = 0$

$r = -1 \,\&\, r = \frac{3}{2}$

$a_2 + a_4 = 2a_3 + 1$

$ar + ar^3 = 2ar^2 + 1$

$a(r + r^3 - 2r^2) = 1$

$a\left(\frac{3}{2}+\frac{27}{8}-\frac{18}{4}\right)=1$

$a=\frac{8}{3}$

When $r=-1, a=-\frac{1}{4}$ (rejected, $a_1 > 0$)

$r=\frac{2}{3}, a=\frac{8}{3}$ (selected)

Now

$a_2 + a_4 + 2a_5$

$=\frac{8}{3}\times\frac{3}{2}+\frac{8}{3}\times\frac{27}{8}+2\times\frac{8}{3}\times\frac{81}{16}$

= 4 + 9 + 27 = 40

28. The integral $\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{(2-x^2)dx}{(2+x^2)\sqrt{4+x^4}}$ is equal to _____.

Ans. 3

Sol. $\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{(2-x^2)}{(x^2+2)\sqrt{4+x^4}}dx$

$$\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{x^2\left(\frac{2}{x^2}-1\right)dx}{x\left(x+\frac{2}{x}\right)\times x\sqrt{\frac{4}{x^2}+x^2}}$$

$$\frac{24}{\pi}\int_0^{\sqrt{2}}\frac{\left(\frac{2}{x^2}-1\right)dx}{\left(x+\frac{2}{x}\right)\times x\sqrt{\left(x+\frac{2}{x}\right)^2-4}}$$

$$x+\frac{2}{x}=t$$

$$dt=\left(1-\frac{2}{x^2}\right)dx$$

$$I=-\frac{24}{\pi}\int\frac{dt}{t\sqrt{t^2-4}}$$

$$=-\frac{24}{\pi}\times\frac{1}{2}\sec^{-1}\left(\frac{x+\frac{2}{x}}{2}\right)\Bigg|_0^{\sqrt{2}}$$

$$=-\frac{12}{\pi}\left[\sec^{-1}\left(\frac{2\sqrt{2}}{2}\right)-\sec^{-1}(\infty)\right]$$

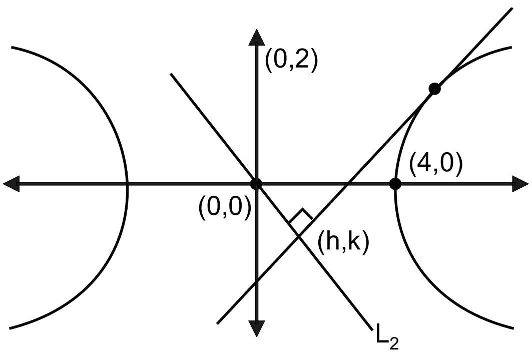
$$=-\frac{12}{\pi}\left[\frac{\pi}{4}-\frac{2\pi}{2\times 2}\right]=-\frac{12}{\pi}\left[-\frac{\pi}{4}\right]$$

= 3

29. Let a line L_1 be tangent to the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ and let L_2 be the line passing through the origin and perpendicular to L_1. If the locus of the point of intersection of L_1 and L_2 is $(x^2+y^2)^2=\alpha x^2+\beta y^2$, then $\alpha+\beta$ is equal to ______.

Ans. 12

Sol.



$$\frac{x\sec\theta}{4}-\frac{y\tan\theta}{2}=1$$

$$m_1=\frac{\sec\theta\times 2}{4(\tan\theta)}=\frac{\sec\theta}{2\tan\theta}$$

$$m_2=\frac{k}{h}$$

$$m_1m_2=-1$$

$$\frac{k}{h}.\frac{\sec\theta}{2\tan\theta}=-1$$

$$\frac{k}{2h\sin\theta}=-1$$

$$\sin\theta=\frac{-k}{2h} \qquad \cos\theta=\frac{\sqrt{4h^2-k^2}}{2h}$$

also

$$\frac{h\sec\theta}{4}-\frac{k\tan\theta}{2}=1$$

$$\frac{h}{4}\frac{2h}{\sqrt{4h^2-k^2}}-\frac{k}{2}\left(\frac{-k}{\sqrt{4h^2-k^2}}\right)=1$$

$$h^2+k^2=2\sqrt{4h^2-k^2}$$

$$(x^2+y^2)^2=4(4x^2-y^2)$$

$$(x^2+y^2)^2=16x^2-4y$$

$\alpha = 16, \beta = -4$

$\alpha + \beta = 16 - 4 = 12$

30. If the probability that a randomly chosen 6-digit number formed by using digits 1 and 8 only is a multiple of 21 is p, then 96 p is equal to _______.

Ans. 33

Sol. $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

Divisible by 21 when divided by 3.

Case – I : All 1 $\rightarrow$ (1)

Case – II : All 8 $\rightarrow$ (1)

Case – III : 3 ones & 3 eights

$$\frac{6!}{3!\times 3!} = 20$$

Required probability $\therefore\ p = \frac{22}{64}$

$$96p = 96 \times \frac{22}{64} = 33$$